

Midterm Review  
Advanced Algebra

Name \_\_\_\_\_

Date \_\_\_\_\_

In this worksheet, you will find at least one problem from each lesson we have covered. Use this as a review of some of the concepts from the year. You should also be going back through old tests, quizzes, and preview sheets, as well as reviewing the lessons you know you struggled with. If you complete the entire set of problems, showing all work, you will get a chance at 5 bonus points on your midterm (same as Chapter Reviews).

1. Evaluate the expression  $\frac{x^2 - 2z}{y}$  when  $x = 4$ ,  $y = -3$ , and  $z = 2$ . (1-1)
2. A cone of radius  $r$  and a height  $h$  has a volume  $V = \frac{1}{3}\pi r^2 h$ . Find the volume in terms of  $\pi$  of a cone with radius 6 inches and height 10 inches. (1-2)
3. State the domain and range of the function  $y = \frac{x+5}{x}$ . (1-2)
4. In Mitarnowskia, electricity costs \$.10819 per kilowatt hour (kwh) for the first 400 kwh and \$.07093 for each kwh over 400. There is also a service charge of \$9.06. Find the amount charged for 725 kwh of electricity. (1-3)
5. Give an example of a function written in Euler notation, then rewrite the function into mapping notation. (1-3)
6. The table below gives the sources for electric power in the United States for the years 1980-1989. Amounts are given in billions of kwh produced. Let  $y$  = the year,  $C(y)$  = the amount produced by coal that year, and  $N(y)$  = the amount produced by nuclear sources that year. (1-4)

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Coal	1162	1203	1192	1259	1342	1402	1386	1464	1541	1551
Nuclear	251	273	283	294	328	384	414	455	527	529

- a. Graph the data on the same set of axes.
- b. Which source of electricity shows a more rapid increase in usage? Justify your answer.

7. Solve and check the equations. (1-5)

a.  $\frac{1}{4}x = 32$

b.  $25 = \frac{5}{6}(m+1)$

c.  $5y + 42 = 3y + 60$

8. When Matt Mitarnowski called Fuzzy Jeff, the call cost \$1.58. If the rate for the call was \$.53 for the first three minutes and \$.15 for each additional minute or fraction thereof, how long did Matt and Jeff talk? (1-5)

9. The formula  $A = P(1+r)^t$  gives the amount  $A$  when a principal  $P$  is invested at a compound interest rate  $r$  for a time of  $t$  years. Solve the formula for  $P$ . (1-6)

10. Young's formula,  $C = \left( \frac{g}{g+12} \right) A$ , tells how much medicine  $C$  to give a child of age  $g$  under age 13 when the adult dosage  $A$  is known. If the dosage for a 12-year-old child is 600 mg, what is the dosage for someone 18 years old? (1-6)

11. Give the first four terms of the sequence  $a_n = \frac{n^2 + n}{2}$ . (1-7)

12. Matt Mitarnowski plans to read 3 books a month to help increase his reading speed. The sequence  $b_n = 3n$  gives the number of books he will have read after  $n$  months. How many books will Matt have read in 8 months? (1-7)

13. The first two terms of a sequence are 2 and 2. Each term after the second is the sum of the previous two terms. (1-8, 1-9)

a. Write a formula for the sequence.

b. Find the next 6 terms.

14. Find the sixth term of the sequence  $\begin{cases} p_1 = 3 \\ p_n = 5p_{n-1}, \text{ for int. } n \geq 2 \end{cases}$ . (1-9)

15. Translate each into an equation. (2-1)

a.  $a$  varies directly as the cube of  $b$ .

b.  $T$  is directly proportional to the fourth power of  $S$ .

16.  $m$  is directly proportional to  $n$ . If  $m = 48$  when  $n = 12$ , find  $m$  when  $n = 3$ . (2-1)

17. Translate each into an equation. (2-2)

a.  $P$  varies inversely with  $d$ .

b.  $e$  is inversely proportional to the cube of  $g$ .

18.  $y$  varies inversely as the square of  $v$ . If  $y = 4$  when  $v = -4$ , find  $y$  when  $v = 8$ . (2-2)

19. Suppose that in a variation problem the value of  $x$  is doubled. How is the value of  $y$  changed if: (2-3)

a.  $y$  varies directly as  $x^3$ ?

b.  $y$  varies inversely as  $x^2$ ?

20. Find the slope of the line through  $(5, -2)$  and  $(3, 8)$ . (2-4)

21. Find the rate of change between  $x = 2$  and  $x = 5$  for  $y = 6x^2$ . (2-5)

22. Graph  $y = \frac{1}{2}x^2$ . (2-5)

23. Find the rate of change between  $x = -2$  and  $x = 2$  for  $y = \frac{8}{x}$ . (2-6)

24. Graph  $y = \frac{8}{x^2}$ . (2-6)

25. While studying gears, a physics class collected the data in the table relating the number of teeth in a gear and the speed of the gear in revolutions per minute. What would be the speed of a gear with 30 teeth? Graph the table and determine an equation to justify your answer. (2-7)

26. A packaging specialist for a toy company compiled the following data. Table I gives the data for the number of racquetballs  $r$  that fit in 30 cm diameter cylindrical cans of height  $h$ . Table II gives data for the number of racquetballs  $r$  that fit in cans 48 cm high with diameter  $d$ . (2-8)

I.

Height of can $h$ (cm)	30	36	48	60	84
Number of racquetballs $r$	100	120	160	200	280

II.

Diameter of a can $d$ (cm)	30	35	40	45	50
Number of racquetballs $r$	160	210	270	340	425

a. Graph the data from each table.

b. How does  $r$  vary with  $h$ ?

c. How does  $d$  vary with  $h$ ?

d. Write an equation relating  $r$ ,  $d$ , and  $h$ , without finding the constant of variation.

27. Translate each into an equation. (2-9)

- a. The area  $A$  of an ellipse varies jointly as its major semiaxis  $b$  and its minor semiaxis  $h$ .
- b. The load  $L$  which will buckle a column is inversely proportional to the square of its length  $l$  and directly proportional to the fourth power of its radius  $r$ .

28.  $a$  varies directly as  $b$  and inversely as the square of  $c$ . When  $b = -8$  and  $c = 2$ ,  $a = -6$ . Find the constant of variation. Then find  $a$  when  $b = 4$  and  $c = -2$ . (2-9)

29. Maggie Brann has 42 computer magazines. She adds two more each month. (3-1)

- a. Write an equation relating the number of magazines  $c$  Maggie has after  $m$  months.
- b. How many magazines are there after 6 months?
- c. After 2 years, will Maggie have 100 computer magazines? Why or why not?

30. Find an equation for the line with slope  $-6$  and  $y$ -intercept  $(0, 9)$ . (3-2)

31. Find an equation for the line with  $y$ -intercept  $(0, 0)$  that is parallel to  $y = 2x - 5$ . (3-2)

32. On a quiz show, 20 points are given for correct answers to regular questions and 50 points are given for correct answers in the bonus round. (3-3)

- a. Write an expression that gives the total number of points earned.
- b. Suppose a contestant earned 650 points. Write an equation for the situation.
- c. Give three different possible solutions to the equation.

33. Graph the following lines. (3-4)

- a.  $8y = 40$
- b.  $3x + 5y = -15$
- c.  $28 = 14x - 7y$

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34. Find equations for each line with the given information. Write your answers in standard form. (3-5)

- a. slope  $-1$  through  $(4, -3)$
- b. through  $(-4, 1)$  parallel to  $4x + 2y = 7$

35. Card Carriers charges \$36 to print 1200 business cards and \$56 for 2700 cards. Assume the relationship between the price and number of cards is linear. (3-5)

- a. Write an equation giving price as a function of the number of cards printed.
- b. Find the set-up cost (cost for printing 0 cards).
- c. Find the cost of printing 6000 cards.

36. The following data give the number of city council members in six cities with various populations. (3-6)

<b>Population</b>	45,000	16,000	320,000	108,000	61,000	176,000
<b>City Council Members</b>	8	7	24	19	12	15

- Find a regression line for the data.
- Use your equation to predict the numbers of city council members in a city with a population of 250,000.
- Find the coefficient of correlation for the data.

37. Write a recursive formula for each sequence. (3-7)

- 17, 28, 39, 50, ...
- $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \dots$

38. Find an explicit formula for each sequence. (3-8)

- 22, -19, -16, -13, ...
- 75, 25, -25, -75, ...

39. Two terms of an arithmetic sequence are given. Write both a recursive and explicit formula for the sequence. (3-8)

$$s_2 = 12 \text{ and } s_8 = 60$$

40. Evaluate each. (3-9)

- $\lfloor 15.7 \rfloor$
- $\lfloor -2.008 \rfloor$
- $\lfloor \sqrt{50} \rfloor$

41. Hot dog buns come in packages of 8. Write an equation that shows the number  $p$  of complete packages that can be made from  $b$  buns. (3-9)

42. Graph the following on a number line. (5-1)

- $x \leq 8$
- $r > -3$
- $7 \geq y \geq -3$

43. Determine whether the ordered pair (3, -10) is a solution to  $\begin{cases} 3x + y = -1 \\ 2x + 3y = -17 \end{cases}$ .

(5-2)

44. Graph the system, state how many solutions there are, and estimate the solutions to the nearest tenth. (5-2)

- $\begin{cases} 2x + 4y = 6 \\ x + y = -1 \end{cases}$
- $\begin{cases} y = \frac{4}{x} \\ y = x \end{cases}$

45. Solve by substitution. (5-3)

a. 
$$\begin{cases} y = x - 7 \\ y = -2x + 5 \end{cases}$$

b. 
$$\begin{cases} xy = -4 \\ x = -4y \end{cases}$$

46. Solve by linear combination. (5-4)

a. 
$$\begin{cases} 8m - 2n = -16 \\ 2m - .5n = -4 \end{cases}$$

b. 
$$\begin{cases} 2a + b - 5c = -21 \\ a + 2b - 2c = -15 \\ a - 4b + c = 18 \end{cases}$$

47. Graph the following. (5-7)

a.  $y > -x - 2$

b.  $-2x - 3y \geq 9$

48. Graph each system and find the vertices of the feasible set. (5-8)

a. 
$$\begin{cases} y > 2x \\ y < -x - 3 \end{cases}$$

b. 
$$\begin{cases} x \geq 0 \\ y \leq 3 \\ y < \frac{1}{2}x + 2 \end{cases}$$

49. Matt Mitarnowski needs to borrow at least \$50,000, some from a credit union and the rest from a bank. At both institutions, he will repay the amount he borrows after 5 years, but he will need to pay simple interest on the loans for each of the five years. He can borrow no more than \$35,000 from the credit union, and he wants the amount borrowed from the bank to be no more than the amount borrowed from the credit union. The credit union charges 11.5% interest and the bank charges 13%. Matt wants to minimize his costs in a single year. (5-9/5-10)

a. Write a system to describe the situation.

b. Graph the system.

c. Find the vertices of the feasible set.

d. Write an equation to be minimized.

e. How much should Matt borrow from each institution?

f. What is the minimum amount of interest Matt will need to pay in one year?